



The Hanbury Brown and Twiss experiment with fermions

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Abstract

We realized an equivalent Hanbury Brown and Twiss experiment for a beam of electrons in a two-dimensional electron gas in the quantum Hall regime. A metallic split gate serves as a tunable beam splitter which is used to partition the incident beam into transmitted and reflected partial beams. The current fluctuations in the reflected and transmitted beam are fully anticorrelated demonstrating that fermions tend to exclude each other (anti-bunching). If the occupation probability of the incident beam is lowered by an additional gate, the anticorrelation is reduced and disappears in the classical limit of a highly diluted beam. © 2000 Elsevier Science B.V. All rights reserved.

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The first quantum statistical measurements were carried out with photons by Hanbury Brown and Twiss (HBT) in the 1950s. In a pioneering experiment HBT determined the size of astronomical radio sources by measuring the spatial coherence of the emitted radiation from correlations between *intensity* fluctuations at two different locations [1]. In a subsequent optical table-top experiment, they tested their idea by measuring intensity correlations for visible light: the light beam of a Hg vapor lamp (IN) was split into a transmitted (T) and a reflected beam (R) (see Fig. 1). The equal-time intensity correlations between the two

separated photon streams was found to be positive (bunching) [2–5]. This is a generic property of particles obeying Bose–Einstein statistics. In contrast to bosons, a negative correlation or ‘antibunching’ is expected for fermions, because fermions have to exclude each other due to the Pauli principle.

In general, the time-dependent number of particles $n(t)$ detected during a certain time interval exhibits fluctuations $\Delta n(t) = n(t) - \langle n \rangle$ around the average $\langle n \rangle$, called shot noise. Shot noise in the electrical current is caused by the discreteness of the electron charge and has been extensively studied in submicrometer-sized nanostructures [6,7]. In a HBT type experiment, not the fluctuations in a single beam are measured, but the correlation between fluctua-

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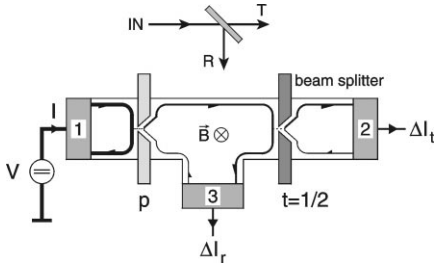


Fig. 1. Intensity correlation experiment for a degenerated beam of electrons realized in a Hall bar connected to four electron reservoirs (shaded). A metallic split gate serves as tunable beam splitter. The incident current can be depleted by an additional gate p . Electrons escaping from contact 1 travel along the upper edge until reaching the beam splitter, where they are either reflected into contact 3 or transmitted to contact 2. Note, that the current incident to the beam splitter equals the current I flowing into contact 1.

tions in the transmitted (T) and reflected (R) beam originating from a beam splitter (see Fig. 1). If the incident beam (IN) is not prepared at a single-particle level, but shows intensity fluctuations Δn around the mean value $\langle n \rangle$, the correlator $\langle \Delta n_t \Delta n_r \rangle$ between the fluctuations in the transmitted Δn_t and the reflected Δn_r beam is given by

$$\langle \Delta n_t \Delta n_r \rangle = t(1-t) \cdot \{ \langle (\Delta n)^2 \rangle - \langle n \rangle \}, \quad (1)$$

where t is the transmission probability of the beam splitter. The cross-correlation in Eq. (1) is a sum of a term proportional to $\langle (\Delta n)^2 \rangle$, which depends on the particle statistics in the incident beam, and a term proportional to $\langle n \rangle$, which is caused by the probabilistic partitioning of single particles at the beam splitter. This second term always gives a negative contribution to the cross-correlation independent of whether the particles are bosons or fermions. The auto-correlation of the transmitted (or reflected) beam is given by

$$\langle (\Delta n_t)^2 \rangle = t(1-t) \cdot \left\{ \frac{t}{1-t} \langle (\Delta n)^2 \rangle + \langle n \rangle \right\}. \quad (2)$$

Here, the sign of the second term is positive in contrast to the cross-correlation in Eq. (1). For an incident beam of particles obeying *Poisson* statistics with $\langle (\Delta n)^2 \rangle = \langle n \rangle$ the cross-correlation is zero (see Eq. (1)). For *super-Poisson* statistics, $\langle (\Delta n)^2 \rangle > \langle n \rangle$, the cross-correlation is positive (HBT result for thermal light). In contrast, anticorrelation results if the noise in the incident beam is *sub-Poissonian* $\langle (\Delta n)^2 \rangle < \langle n \rangle$.

Maximal anticorrelation is obtained if the incident beam carries no fluctuations ($\langle (\Delta n)^2 \rangle = 0$), which is the case for a completely degenerated electron beam at zero temperature. Noise suppression in a single beam according to the Pauli principle has recently been found in electrical measurements on quantum-point contacts and nanowires [8–13].

A HBT-type intensity correlation experiment for particles with non-classical Fermi–Dirac statistics has not yet been carried out. Attempts to measure the expected negative correlation in a beam of free electrons have not been successful yet, mainly because of the low particle density in such a beam. The HBT experiment for fermions has recently been carried out in two independent experiments based on semiconductor devices [14,15]. Theoretically, these experiments and other multiterminal correlation experiments have been considered before [6,16–21].

One way to realize a HBT experiment with electrons has been proposed by Büttiker [6]: In a two-dimensional electron gas (2 DEG) in the quantum Hall regime the current flows in one-dimensional channels along the edges of the device (see Fig. 1). These edge channels can be used to separate the incident from the reflected beam. If there would be no magnetic field the current would also flow in the bulk and incident and reflected beam could not be distinguished. The beam splitter is a lithographically patterned metallic split gate, which can be tuned by an applied voltage. In addition, there is a second gate with transmission probability p before the beam splitter which is used to dilute the occupation in the incident beam. Applying a constant voltage V to contact 1 the charge current I is injected into the Hall bar. The magnetic field perpendicular to the 2 DEG is adjusted to filling factor $\nu = 2$, so that the current flows in one spin-degenerated edge state. Let us first consider the case, where the additional gate p is not used: the electrons escaping from contact 1 travel along the upper edge until reaching the beam splitter, where they are either transmitted with probability t to leave the device at contact 2, or reflected with probability $r = 1 - t$ leaving at contact 3. Provided $eV \gg kT$, the theory predicts for the spectral densities of the auto- and cross-correlation according to Eqs. (1) and (2) [17]

$$\langle \Delta I_\alpha \Delta I_\beta \rangle_\omega = \pm 2e|I|t(1-t) \quad (3)$$

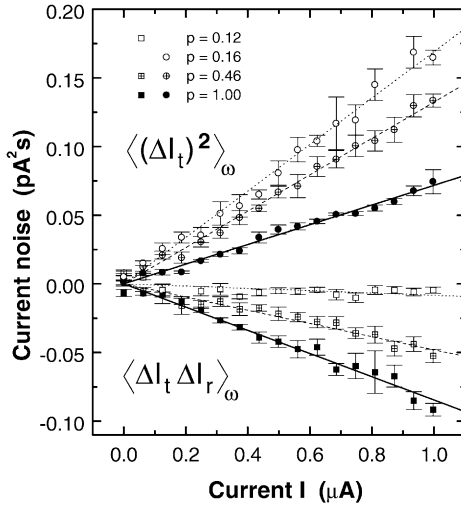


Fig. 2. Measured spectral densities of correlations between current-fluctuations as a function of the current I of the incident beam at $T = 2.5$ K. $\langle \Delta I_t \Delta I_r \rangle_\omega$ denotes the cross-correlation between the transmitted and reflected beams and $\langle (\Delta I_t)^2 \rangle_\omega$ the auto-correlation in the transmitted beam. The offset noise arising from thermal fluctuations and residual amplifier noise has been subtracted. The absolute slopes are $(0.23 \pm 0.030) \cdot 2e$ and $(0.26 \pm 0.037) \cdot 2e$ for the auto-correlation and cross-correlation, respectively, in agreement with the expected prefactor $t(1-t) = \frac{1}{4}$. If the incident beam is diluted by lowering the transmission probability p , the cross-correlation gets smaller and disappears for p close to zero. The auto-correlation itself increases, because the incident beam is noisy.

with e the electron charge and α, β either t or r . The positive sign corresponds to the auto-correlation, where $\alpha = \beta$, and the negative one to the cross-correlation with $\alpha \neq \beta$. Because the cross-/auto-correlation is largest for $t = \frac{1}{2}$ the beam splitter is adjusted to transmit and reflect electrons with 50% probability.

Fig. 2 shows the cross-correlation $\langle \Delta I_t \Delta I_r \rangle_\omega$ (solid squares) of the fluctuations ΔI_t and ΔI_r versus bias current I at $T = 2.5$ K. A nearly linear dependence with a negative slope is found showing that the fluctuations are indeed anticorrelated. The auto-correlation (solid circles) of the transmitted current $\langle (\Delta I_t)^2 \rangle_\omega$ (or reflected current, not shown) has a positive slope. The negative cross-correlation and the positive auto-correlation are equal in magnitude confirming that the partial beams are fully anticorrelated. We can therefore conclude that there is no uncertainty in the occupation of the incident beam, that is $\langle (\Delta n)^2 \rangle = 0$ in Eqs. (1) and (2). All states in the incident beam are

occupied with probability one and hence are noiseless by virtue of the Pauli principle. Formally, this follows also from $\langle (\Delta n)^2 \rangle = \langle (\Delta n_t + \Delta n_r)^2 \rangle = \langle (\Delta n_t)^2 \rangle + 2\langle \Delta n_t \Delta n_r \rangle + \langle (\Delta n_r)^2 \rangle = 0$ within experimental accuracy. The fact that the current I of the incident beam is noiseless demonstrates that the constant voltage applied to reservoir 1 is converted into a *constant* current $e^2 V/h$ (per accessible mode) according to the fundamental requirement of the Landauer–Büttiker formalism.

In an extension of our experiment we have changed the statistics in the incident beam using the additional gate with transmission $p \in [0, 1]$. If $\langle n \rangle$ is the mean particle number of the beam incident to the beam splitter, the particle number behind the first gate p is given by $\langle \tilde{n} \rangle = p \langle n \rangle$ with noise $\langle (\Delta \tilde{n})^2 \rangle = p(1-p) \langle n \rangle$. Using Eqs. (1) and (2) the normalized auto- and cross-correlation depend on the transmission probability p as

$$\frac{\langle (\Delta I_t)^2 \rangle_\omega}{2eI} = t(1-pt) = \frac{2-p}{4}, \quad (4)$$

$$\frac{\langle \Delta I_t \Delta I_r \rangle_\omega}{2eI} = -t(1-t)p = -\frac{p}{4}, \quad (5)$$

for $t = \frac{1}{2}$. If p decreases from 1 to 0 the states in the incident beam are diluted and the anticorrelation (Eq. (5)) becomes smaller ('+centered' and open squares in Fig. 2). In case of very low transmission p the statistics in the incident beam is Poissonian and the anticorrelation disappears as discussed above. The auto-correlation itself increases ('+centered' and open circles in Fig. 2), because of the noise in the incident beam (see Eq. (2)). The dependence of the auto- and cross-correlation on the probability p is shown in Fig. 3. The measured slopes of the data in Fig. 2 are in good agreement with the predictions of Eqs. (4) and (5) within experimental accuracy.

In conclusion, a HBT-type experiment with electrons has been realized in a solid state device. We demonstrate that current correlations are sensitive to the particle statistics in the incident beam. Full anticorrelation is observed for electrons obeying Fermi–Dirac statistics, whereas the anticorrelation is gradually suppressed if the incident beam is diluted. It would be interesting to extend this kind of experiments to electronic states obeying different statistics like the fractional quantum Hall state [22,23]. The observation of

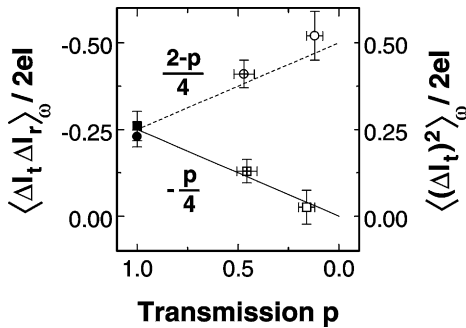


Fig. 3. Dependence of the auto- and cross-correlation on the transmission probability p of the additional gate for $t = \frac{1}{2}$. If p is lowered from 1 to 0 the statistics in the incident beam are changed from Fermi–Dirac to Poisson statistics. The normalized auto-correlation $\langle (\Delta I_t)^2 \rangle_\omega / 2eI$ increases from $\frac{1}{4}$ to $\frac{1}{2}$ and the normalized cross-correlation $\langle \Delta I_t \Delta I_r \rangle_\omega / 2eI$ from $-\frac{1}{4}$ to 0 in good agreement with the prediction of Eqs. (4) and (5).

bunching of electrons might be possible by preparing electronic states with fluctuations larger than the classical Poisson value, which have recently been observed in resonant tunneling devices and in superconducting weak links [24–27].

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