

SHOT NOISE OF SERIES QUANTUM POINT CONTACTS

S. OBERHOLZER¹, E. V. SUKHORUKOV², C. STRUNK³, C. SCHÖNENBERGER¹,

¹*University of Basel, Klingelbergstr. 82, CH-4056 Basel, Switzerland*

²*University of Florida, Gainesville, Florida 32611-8440, USA*

³*University of Regensburg, D-93040 Regensburg, Germany*

Shot noise of series quantum point contacts forming a sequence of cavities in a two dimensional electron gas are studied theoretically and experimentally. Noise in such a structure originates from local scattering at the point contacts as well as from chaotic motion of the electrons in the cavities. The observed shot noise allows to distinguish between noise of the point contacts and noise from the cavities in reasonable agreement with theoretical predictions.

1 Introduction

Shot noise is a non-equilibrium type of electrical current noise directly resulting from random transfer of discrete charge quanta¹. For Poissonian transfer of single electrons the spectral density of the current fluctuations is $S_{Poisson} = 2e|I|$. Correlations imposed by fermionic statistics or Coulomb interaction may change shot noise from $S_{Poisson}$ which is expressed by the Fano factor F defined as $F \equiv S/S_{Poisson}$. As an example, a one mode quantum wire with an intermediate barrier of transmission probability T has a Fano factor of $F = 1 - T$. Thus, for $T > 0$ shot noise is suppressed below its full Poissonian value. In various mesoscopic systems universal Fano factors have been found such as $F = 1/3$ in metallic diffusive wires^{2,3} or $F = 1/4$ in chaotic cavities⁶. This paper is devoted to the crossover between shot noise of a single scatterer with $F = 1 - T$ to the diffusive regime with a large number of scatterers for which F is $1/3$. Experimentally, this can be investigated measuring shot noise of several quantum point contacts (QPC), which model impurity scattering in a diffusive wire.

2 Theory

Shot noise for a sequence of N planar tunnel barriers has been calculated by de Jong and Beenakker within a semiclassical description based on the Boltzmann-Langevin approach⁷. For equal transmission probabilities $T_{1,2,\dots,N} = T$ the Fano factor F reaches $1/3$ with increasing barrier number ($N \rightarrow \infty$) for any value of the transparency $T \in [0, 1]$. Hence, in this description a diffusive conductor can be modeled as the continuum limit of a series of tunnel barriers. However, the calculation in Ref.⁷ is only valid for one-dimensional transport since it neglects transverse motion of the electrons in between the barriers. It is therefore not appropriate to describe our physical system consisting of a series of quantum point contacts. Here, we consider the case that there are cavities between the barriers in which the electrons scatter chaotically leading to additional cavity noise^{4,5}.

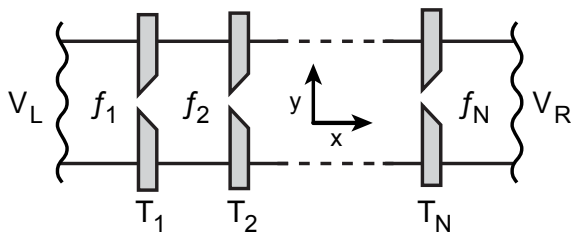


Figure 1: Schematic of the considered system: N quantum point contacts forming a series of cavities. $f_{n=1,\dots,N}$ denote the distribution functions of the electrons. In such a system shot noise arises due to quantum diffraction inside the cavity as well as to partitioning at the contacts.

In general, the fluctuations in the total current through a system as shown in Fig. 1 can be written as⁵

$$\delta I = \delta I_n^S + G_n(\delta V_{n-1} - \delta V_n) \quad n = 1, \dots, N \quad (1)$$

using the fact that the total current is conserved. $G_n = G_0 \sum_k T_{kn}$ ($G_0 = 2e^2/h$) is the conductance of the n -th barrier and δV_n the voltage fluctuations inside the n -th cavity. δI_n^S are the ‘‘Poissonian’’ current fluctuations of a single QPC⁸:

$$\langle \delta I_n^S \delta I_m^S \rangle = S_n \delta_{nm} \quad \text{with} \quad (2)$$

$$S_n = 2G_0 \sum_k \int d\epsilon \left[T_{kn} f_{n-1} (1 - f_{n-1}) + T_{kn} f_n (1 - f_n) + T_{kn} (1 - T_{kn}) (f_{n-1} - f_n)^2 \right]. \quad (3)$$

Summing Eq. 1 over n while assuming equal conductances $G_n = G$ the total noise power follows as

$$S \equiv \langle \delta I^2 \rangle = \frac{1}{N^2} \sum_{n,m=1}^N \langle \delta I_n^S \delta I_m^S \rangle = \frac{1}{N^2} \sum_{n=1}^N S_n, \quad (4)$$

where we assumed $\delta V_1 = \delta V_N = 0$, i.e. no fluctuations in the potential of the perfect metallic leads.

2.1 Non-interacting electrons

The distribution function f_n of the n -th cavity follows from the conservation of electrons in each energy interval⁵

$$f_n(\epsilon) = \left(\frac{N-n}{N} \right) f_L(\epsilon) + \frac{n}{N} f_R(\epsilon), \quad (5)$$

with $f_L(\epsilon) = f_F(\epsilon, eV_L, \theta)$ and $f_R(\epsilon) = f_F(\epsilon, eV_R, \theta)$ the equilibrium Fermi function in the left and right reservoir, respectively. θ denotes the bath temperature. For simplicity we consider in the following only one propagating mode and introduce the backscattering parameter $\mathcal{R}_n \equiv 1 - T_n = \mathcal{R}$ assumed to be the same for all barriers. Substituting the distribution function f_n (Eq. 5) into Eq. 3, the total noise of N point contacts in series follows from Eq. 4. In the zero temperature limit we obtain for the Fano factor

$$F \equiv \frac{S}{2e|I|} = \left(\frac{1}{3} + \frac{\mathcal{R}}{N^2} - \frac{1}{3N^2} \right). \quad (6)$$

For a single QPC ($N = 1$) F equals the backscattering parameter $\mathcal{R} = 1 - T$ as expected. For $N = 2$ we have a single cavity which is separated from the leads by two QPCs and for ideal contacts ($\mathcal{R} = 0$) the Fano factor is $1/4$. If the QPCs are in tunneling regime ($\mathcal{R} \approx 1$) noise is dominated by the QPCs and the dynamics inside the cavity play no role. The Poissonian voltage noise of the two contacts adds up resulting in a Fano factor $1/2$. In the intermediate regime $F = \frac{1}{4}(1 + \mathcal{R})$. Increasing the number of QPCs ($N \rightarrow \infty$) the Fanofactor F reaches $1/3$ independent of $\mathcal{R} = 1 - T$ as for the calculation in Ref.⁷.

In Fig. 2(a) we compare our result of Eq. 6 with the result for one-dimensional tunnel-barriers. For point contacts with low transparencies ($T = 0.1$) the results are very similar because in this case noise is dominated by the contacts. But for high transparencies ($T = 0.9$) the calculation including noise of cavities (Eq. 6) shows that shot noise increases much faster with the number of QPCs N in contrast to the linear model of de Jong and Beenakker⁷.

2.2 Interacting electrons

In case of strong electron-electron interaction in the cavities the distribution function inside the n -th cavity $f_n(\epsilon, \theta)$ equals a Fermi function $f_F(eV_n, \theta_n)$ at an elevated electron temperature

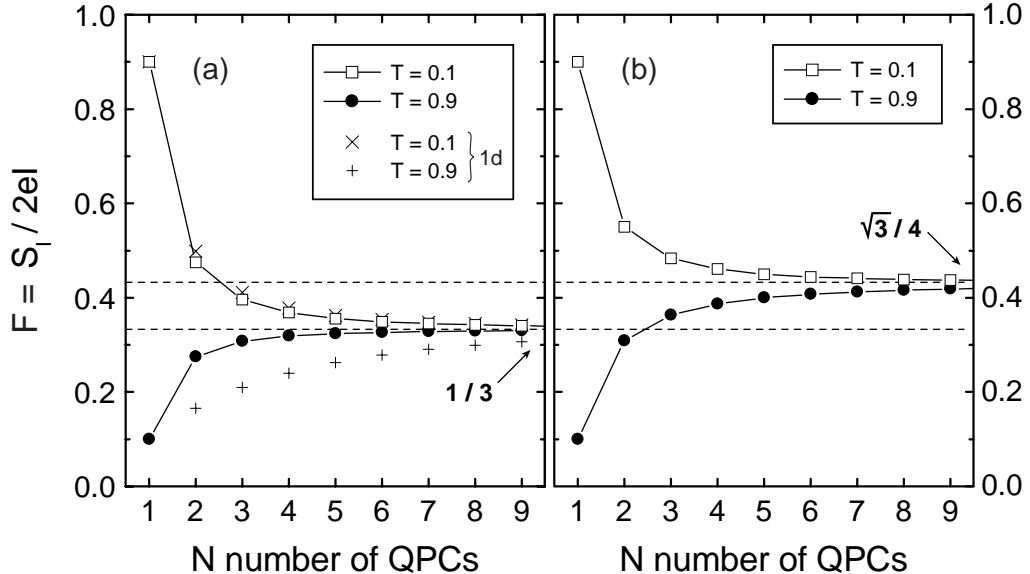


Figure 2: Shot noise normalized to the Poissonian limit $2e|I|$ as a function of the number of QPCs N in series for cold (a) and hot electrons (b) at zero bath temperature. The crosses ($\times, +$) in the left plot correspond to a calculation of de Jong and Beenakker for one-dimensional transport, not including the noise arising from chaotic motion of the carriers in the cavities between the barriers.

θ_n . $V_n = V(1 - n/N)$ is the potential in the n -th cavity with $V = V_L - V_R$ the potential difference between left and right reservoir. The electron temperature θ_n follows from energy balance equation using the Wiedemann-Franz law⁹:

$$\theta_n^2 = \theta^2 + \frac{3n(N-n)}{N^2} \left(\frac{eV}{\pi k_B} \right)^2. \quad (7)$$

Using Eq. 3 and 4 the noise power for hot electrons can be calculated numerically. Results are shown in Fig. 2(b). In the continuum limit ($N \rightarrow \infty$) the Fano factor F equals $\sqrt{3}/4$ as for a diffusive wire with electron heating^{10,11}.

3 Experimental

Experimentally, a series of up to four QPCs are defined by split gates across a Hall bar in a two dimensional electron gas (2 DEG) forming at the interface of a standard GaAs/GaAlAs-heterojunction (Fig. 3(a)). In the experiment, we used only up to three QPCs because one of the four did not show proper conduction quantization. Voltage fluctuations across the series of QPCs are detected using a crosscorrelation technique with two identical low-noise amplifiers. A spectrum analyzer calculates the crosscorrelation of the two amplified signals. Voltage noise is measured as a function of current for one to three QPCs in series with fixed transmission probabilities. The transmission of a single point contact can be independently adjusted measuring its conductance $G = G_0 \sum_k T_k$. The spectral density of the voltage fluctuations are averaged over a frequency bandwidth of 1 kHz at ~ 8 kHz. For experimental details, see Ref.^{3,6}.

4 Results and discussion

In Fig. 3(b) the Fano factor $F \equiv S/2e|I|$ extracted from the shot noise measurements is plotted as a function of the number of point contacts. The black dots correspond to experimental

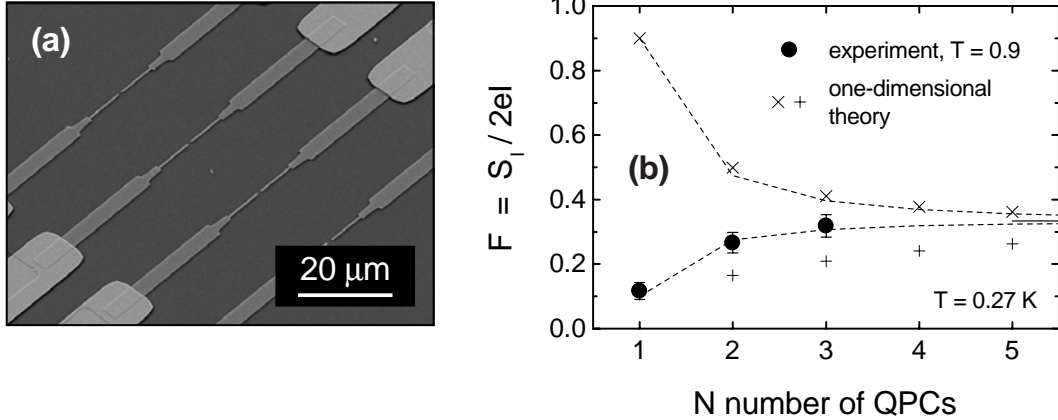


Figure 3: (a) Scanning electron microscope picture of the measured device. The Fermi energy of the 2 DEG is 94 K and the elastic mean free path $l_e \approx 7 \mu\text{m}$. (b) The black points are experimental data for one mode transmitted at the point contacts with probability 0.9. The dashed line are theoretical predictions including cavity noise for non-interacting electrons. The crosses correspond to the one-dimensional model of de Jong and Beenakker.

data obtained for a transmission probability $T = 0.9$ of the single QPCs. The dashed line is the prediction of Eq. 6, whereas the crosses correspond to the one-dimensional calculation of Ref. 7. For $N = 1$ shot noise is strongly suppressed compared to $S_{Poisson} = 2e|I|$, as expected for a single QPC with high transmission probability¹². When N goes from 1 to 3 shot noise becomes larger. Because in our system we not only have partition noise at the contacts, but also additional cavity noise the Fano factor indeed increases much faster with increasing number of the contacts N compared to the prediction of the one-dimensional theory. Since the cavities are huge compared to the mean free path of the electrons the electrons stay relatively long in the cavity so that they might inelastically scatter from each other. However, the uncertainties of the experimental data do not allow to distinguish between the cold- and hot-electron regime.

Acknowledgments

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