

The amplitude of non-equilibrium quantum interference in metallic mesoscopic systems

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Abstract. – We study the influence of a DC bias voltage V on quantum interference corrections to the measured differential conductance in metallic mesoscopic wires and rings. The amplitude of both universal conductance fluctuations (UCF) and Aharonov-Bohm effect (ABE) is enhanced several times for voltages larger than the Thouless energy. The enhancement persists even in the presence of inelastic electron-electron scattering up to $V \sim 1$ mV. For larger voltages electron-phonon collisions lead to the amplitude decaying as a power law for the UCF and exponentially for the ABE. We obtain good agreement of the experimental data with a model which takes into account the decrease of the electron phase-coherence length due to electron-electron and electron-phonon scattering.

If the size of a conductor is of the order of the electron phase-coherence length $L_\varphi(T)$, the wave character of electrons leads to experimentally observable quantum interference contributions to the conductance G . These are the aperiodic and periodic fluctuations δG of the conductance around its average value. In the former case, called universal conductance fluctuations, the interference pattern is formed as a superposition of contributions from a continuous range of possible interference paths. Averaged over either impurity configuration, magnetic field or energy, the root-mean-square (rms) conductance fluctuation δG_{rms} is of the order $\sim e^2/h$ [1]. The periodic fluctuations, known as the Aharonov-Bohm effect [2], are observed if the interference is imposed by geometry of a sample, most commonly in the form of a loop. If the loop is threaded by a magnetic flux ϕ , $\delta G(\phi)$ exhibits periodic oscillations with a period h/e , and δG_{rms} is again $\sim e^2/h$. Both the UCF [3] and the ABE [4] are suppressed by ensemble averaging if independent phase-coherent units are connected in series. The above behaviour is characteristic of the linear-response regime, *i.e.* of $eV \ll k_B T$ or $eV \ll E_c$, where E_c is the coherence energy (Thouless energy) determined by the size of the conductor. In non-equilibrium ($eV \gg E_c, k_B T$) the fluctuations are expected to be remarkably different, as predicted theoretically by Larkin and Khmel'nitskiĭ (LK) [5]. If inelastic processes can be

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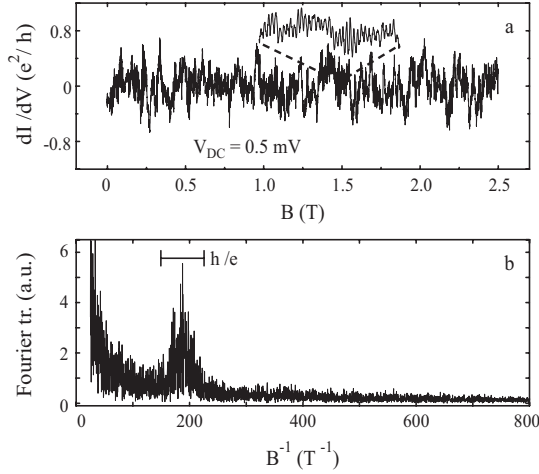


Fig. 1 – (a) A differential-magnetoconductance trace of sample S_{ABE} , taken at $V_{\text{DC}} = 0.5$ mV. Both the periodic (expanded view) and aperiodic fluctuations are present. (b) Fourier transform of (a), showing a well-defined peak at the position corresponding to $\phi = h/e$.

neglected the rms fluctuation δg_{rms} of the *differential conductance* g increases with V according to $\delta g_{\text{rms}} \sim (e^2/h)\sqrt{V/V_c}$, where $V_c = E_c/e$. This at first sight surprising result can be understood as follows. At $V \gg V_c$ the relevant energy range for the transport subdivides into $N = V/V_c$ uncorrelated energy intervals, each contributing to the fluctuations of the current by an amount $\sim (e^2/h)V_c$. Incoherent superposition of these contributions leads to δg_{rms} being $N^{1/2}$ times larger than e^2/h . Inelastic scattering at large voltages destroys quantum interference. The enhancement of $\delta g_{\text{rms}}(V)$ is thereby suppressed and δg_{rms} eventually decreases with increasing voltage [5]. Experimental studies of both the UCF [6–9] and ABE [6, 10, 11] under non-equilibrium conditions have been done, but did not result in a satisfactory understanding of $\delta g(V)$. Moreover, [8] reported on a voltage-independent δg_{rms} , [6] on a decrease of $\delta g_{\text{rms}}(V)$ for $V > V_c$, whereas in [9–11] it was found that $\delta g_{\text{rms}}(V)$ shows a non-monotonic behaviour. We also note that explanations [10, 11] of the non-monotonic behaviour of $\delta g(V)$, observed in the present work as well, were at a qualitative level.

In this paper we report on the non-equilibrium UCF and ABE in diffusive gold samples measured over a wide voltage range of $V \gg k_B T/e, V_c$ (strongly V -dominated energy averaging) that covers both the low-voltage $(V/V_c)^{1/2}$ enhancement of δg_{rms} and its suppression at large voltages. Emphasis is put on the decay of $\delta g_{\text{rms}}^{\text{UCF}}(V)$ and $\delta g_{\text{rms}}^{\text{ABE}}(V)$. It is shown that $\delta g_{\text{rms}}(V)$ decays as a power law for the UCF and exponentially for the ABE. A quantitative comparison between simple models and the experiments allows to extract the voltage-dependent phase-coherence length $L_\varphi(V)$ and to discuss the nature of inelastic scattering processes out of equilibrium.

The samples were produced by electron-beam lithography and evaporation of 99.99% pure gold. The substrate was silicon covered with 400 nm of SiO_2 . Three 20 nm thick samples of different planar geometries were prepared [12]: 1) for the ABE measurements a ring of average diameter 1 μm and line width 0.09 μm (sample S_{ABE}), with a resistance of 25.8 Ω at 0.3 K; 2) for the UCF measurements a $L_w = 1.5$ μm long and 0.13 μm wide wire (sample S_{UCF}), with a resistance of 18.4 Ω at 0.3 K; 3) for the weak-localisation (WL) measurements a 98 μm long and 0.17 μm wide wire (sample S_{WL}). Sample S_{WL} was made relatively long in order

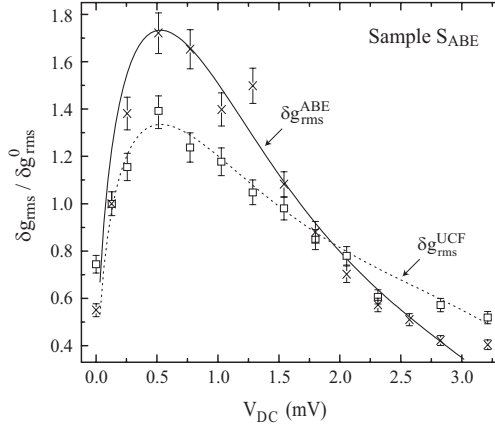


Fig. 2 – Plot of $\delta g_{\text{rms}}^{\text{ABE}}$ (crosses) and $\delta g_{\text{rms}}^{\text{UCF}}$ (open squares) *vs.* V_{DC} for sample S_{ABE} , with δg_{rms} normalised to the values δg_{rms}^0 at the first $V_{\text{DC}} \neq 0$ points (which also satisfy the criterion $V_{\text{DC}} \gg V_{c0}$). The lines are plots of eq. (1) (dotted line) and eq. (2) (solid line) discussed in the text.

to suppress the UCF. All samples were produced under identical conditions (the same source of gold and the same evaporation parameters). The diffusion constants D are consequently very similar: $116 \text{ cm}^2/\text{s}$ for samples S_{UCF} and S_{WL} , and $110 \text{ cm}^2/\text{s}$ for sample S_{ABE} . The measurements were carried out in a ^3He cryostat with cryogenic rf-filtering, using a low-frequency (37 Hz) lock-in technique to measure the differential conductance. Typical voltage resolution was $\sim 0.3 \text{ nV}$. Non-equilibrium UCF and ABE were measured at $T = 300 \text{ mK}$ by superimposing a comparatively large DC bias voltage V_{DC} to the small excitation voltage V_{AC} . The following hierarchy of energies was always maintained: $eV_{\text{DC}} \gg k_{\text{B}}T \geq eV_{\text{AC}} > E_{\text{c}}$. Ensemble averaging was achieved by measuring the UCF and ABE over a magnetic-field range of $\sim 2 \text{ T}$, largely exceeding the correlation field. Typical sweep rates were 0.1 mT/s . Figure 1a displays raw data of a differential-conductance measurement on sample S_{ABE} , taken at $V_{\text{DC}} = 0.5 \text{ mV}$. Both types of the fluctuations are present. In fig. 1b we show the Fourier transform of the same data, exhibiting a well-defined peak at the position corresponding to $\phi = h/e$. From the magnetoconductance traces we have extracted $\delta g_{\text{rms}}^{\text{ABE}}$ and $\delta g_{\text{rms}}^{\text{UCF}}$ for sample S_{ABE} , as shown in fig. 2 (discussed in more detail later), and $\delta g_{\text{rms}}^{\text{UCF}}$ for sample S_{UCF} . The values of $\delta g_{\text{rms}}^{\text{UCF}}$ have been determined as standard deviations of the whole differential-conductance traces. The amplitudes $\delta g_{\text{rms}}^{\text{ABE}}$ have been calculated by averaging the periodic part of the fluctuations over ~ 500 periods, after the coarse background has been removed numerically.

The WL was measured on sample S_{WL} in the linear-response regime ($V_{\text{DC}} = 0$) and in the temperature range $0.3\text{--}10 \text{ K}$. By fitting a one-dimensional WL expression [13] to the low-field magnetoconductance data we obtained the linear response L_{φ} as a function of temperature, as shown in the inset to fig. 3. Similarly to published work [14, 15], $L_{\varphi}(T)$ saturates at low temperatures (below 1 K), which has been attributed to impurity-mediated inelastic electron-electron scattering [16]. At high temperatures L_{φ} follows a $L_{\varphi} \propto T^{-q}$ -dependence with $q \approx 1.2$ that suggests dephasing by electron-phonon interaction. At $T = 300 \text{ mK}$, $L_{\varphi} = 3 \text{ }\mu\text{m}$ and $\tau_{\varphi} = \sqrt{L_{\varphi}^2/D} = 0.77 \text{ ns}$, implying that in the linear-response regime the diffusion of electrons is coherent throughout the samples S_{UCF} and S_{ABE} . L_{φ} has to be compared further with the effective length of the sample L_{c0} which depends on the particular geometry and the coherence

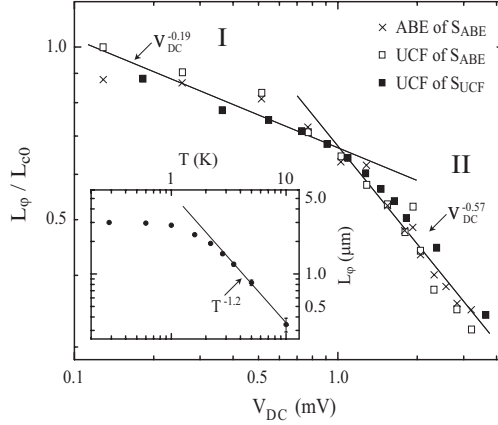


Fig. 3 – Phase-coherence length $L_\varphi(V_{\text{DC}})$, normalised to the characteristic lengths L_{c0} defined in the text, deduced from the non-equilibrium UCF and ABE measurements. Crosses: ABE in sample S_{ABE} . Open squares: UCF in sample S_{ABE} . Full squares: UCF in sample S_{UCF} . Below $V_{\text{DC}} \sim 1$ mV, $L_\varphi \propto V_{\text{DC}}^{-0.19 \pm 0.02}$, and above $L_\varphi \propto V_{\text{DC}}^{-0.57 \pm 0.03}$. Inset: linear-response coherence length $L_\varphi(T)$ found from the WL measurements on sample S_{WL} .

phenomenon investigated. For the UCF in sample S_{UCF} , $L_{c0} = L_w = 1.5 \mu\text{m}$. In the case of ABE in sample S_{ABE} , $L_{c0} = C_r \approx 3.14 \mu\text{m}$, where C_r is the circumference of the ring. Finally, for the UCF in sample S_{ABE} , L_{c0} is calculated as the distances ring-voltage contacts plus $C_r/2$, which gives $\approx 2.2 \mu\text{m}$. Thus, $L_\varphi(300 \text{ mK})$ is always bigger or very similar to L_{c0} , so that the linear-response coherence energy is set by the time of electron diffusion through the sample according to $E_{c0} = eV_{c0} = \hbar D/L_{c0}^2$.

With increasing temperature or applied voltage the coherence length decreases, and once it becomes shorter than L_{c0} the interference contributions are suppressed. The effect of temperature was demonstrated by Milliken *et al.* [17]. Their method was to determine $L_\varphi(T)$ from the WL and to use this to describe the decay of *equilibrium* δG , which turned out to be a power law for the UCF and exponential for the ABE. Our approach is complementary: we keep the bath temperature constant and investigate the voltage dependence of the *non-equilibrium* δg . While Milliken *et al.* observed a monotonically decreasing $\delta G(T)$, in our case $\delta g(V)$ is a non-monotonic function, as shown in fig. 2. For small V_{DC} (but still much larger than V_{c0}), δg_{rms} is enhanced by V_{DC} . At higher voltages δg_{rms} decreases with V_{DC} , *faster for the ABE than for the UCF*, which is in qualitative agreement with the results of Milliken *et al.*

Let us first turn to the details of non-equilibrium UCF. For convenience, we denote the phase-coherence length at a finite bias voltage by $L_\varphi(V)$, while $L_\varphi(T)$ refers to the linear-response value. If $V_{\text{DC}} \gg V_{c0}$, $k_B T/e$ and $L_\varphi(V_{\text{DC}}) > L_{c0}$, the LK enhancement of δg_{rms} holds and one expects that $\delta g_{\text{rms}} \propto \sqrt{V_{\text{DC}}/V_{c0}}$. Once $L_\varphi(V_{\text{DC}})$ becomes substantially smaller than L_{c0} , inelastic processes start to be important for most of the electrons, and the sample effectively subdivides into L_{c0}/L_φ uncorrelated phase-coherent sections in series. The LK enhancement in a single section is still valid if the voltage drop $V_{\text{DC}}^{\text{sec}} = V_{\text{DC}}(L_\varphi/L_{c0})$ over a section exceeds the corresponding coherence voltage $V_c^{\text{sec}} = V_{c0}(L_{c0}/L_\varphi)^2$. Following the LK arguments, δg_{rms} corresponding to a single section is proportional to $(V_{\text{DC}}^{\text{sec}}/V_c^{\text{sec}})^{1/2} = [(V_{\text{DC}}/V_{c0})(L_\varphi/L_{c0})^3]^{1/2}$. Incoherent addition of these contributions of sections in series leads

to the *differential conductance* being suppressed by a factor $(L_\varphi/L_{c0})^{1/2}$, and hence

$$\delta g_{\text{rms}}^{\text{UCF}} = A_{\text{UCF}} \frac{e^2}{h} \sqrt{\frac{V_{\text{DC}}}{V_{c0}}} \left(\frac{L_\varphi(V_{\text{DC}})}{L_{c0}} \right)^2, \quad (1)$$

where A_{UCF} is a constant prefactor. This expression is different from that resulting from the linear-response ensemble averaging, and valid if $V_{\text{DC}} > V_{c0}$ and $L_\varphi < L_{c0}$ [18]. It enables an unambiguous determination of $L_\varphi(V_{\text{DC}})$. Since the form of $L_\varphi(V_{\text{DC}})$ is not known *a priori*, we cannot determine A_{UCF} and $L_\varphi(V_{\text{DC}})$ simultaneously from a direct fit to the data. To avoid this problem we fix A_{UCF} by setting $L_\varphi = L_{c0}$ for the first measured point which satisfies $V_{\text{DC}} \gg V_{c0}$, and then simply extract $L_\varphi(V_{\text{DC}})$ for all the other points from the measured data. For the two sets of UCF data, *i.e.* for samples S_{UCF} and S_{ABE} , we have obtained excellent agreement in $L_\varphi(V_{\text{DC}})$, as we show in fig. 3 by open (S_{ABE}) and full (S_{UCF}) squares. Normalised to the corresponding values of L_{c0} , both results collapse onto a single curve. The phase-coherence length deduced from our non-equilibrium experiment $L_\varphi(V)$ has a qualitatively similar functional form as $L_\varphi(T)$ obtained from the WL measurements (apart from the saturation of $L_\varphi(T)$, since saturation of $L_\varphi(V)$ is not expected for $eV \gg k_{\text{B}}T$ [16]). Two regimes with different power law dependences are discernible. In the region of increasing $\delta g_{\text{rms}}(V_{\text{DC}})$ there is a power law $L_\varphi(V_{\text{DC}}) \propto V_{\text{DC}}^{-s}$, where $s = 0.19 \pm 0.02$. For decreasing $\delta g_{\text{rms}}(V_{\text{DC}})$, *i.e.* where $L_\varphi(V_{\text{DC}})$ is considerably smaller than L_{c0} , the dependence changes to $L_\varphi \propto V_{\text{DC}}^{-p}$ with $p = 0.57 \pm 0.03$. Inserting thus found L_φ back into eq. (1) results in the dotted line in fig. 2.

We now discuss the non-equilibrium ABE. Here the LK theory cannot be directly applied to extract δg_{rms} . Subdivision of a ring into phase-coherent sections makes no sense in this case, as only those electrons which stay coherent over the whole length C_{r} contribute to the interference. However, since the same physics governs the UCF and ABE, $\delta g_{\text{rms}}^{\text{ABE}}$ is expected to be $\propto \sqrt{V_{\text{DC}}/V_{c0}}$ in the regime where inelastic scattering is absent. This is indeed the case, as shown in fig. 2. The form of the suppression of $\delta g_{\text{rms}}^{\text{ABE}}$ can be inferred from a theoretical analysis of DiVincenzo and Kane [19]. They have shown that inelastic processes influence the ABE in two ways. First, the fluctuations are suppressed as $\exp[-\beta C_{\text{r}}/L_\varphi]$, where β is of order unity in the range $1 < C_{\text{r}}/L_\varphi < 3$ and smaller in the case of a true exponential decay occurring in the asymptotic limit $C_{\text{r}}/L_\varphi \rightarrow \infty$. Second, the proper form of E_{c} in the presence of inelastic processes is given by $E_{\text{c}}^{\text{in}} = \hbar D/C_{\text{r}}^{2-\alpha} L_\varphi^\alpha = E_{\text{c0}}(C_{\text{r}}/L_\varphi)^\alpha$ with $\alpha \approx 1.3$. This renormalisation of the coherence energy is a consequence of the statistics for those electrons that diffuse around a ring without being scattered inelastically [19]. Roughly, the probability distribution for an electron to contribute to the ABE has a maximum at a value of the electron traversal time $\sim D/C_{\text{r}} L_\varphi$ [19]. Combining the above arguments we can write for $L_\varphi < C_{\text{r}}$

$$\delta g_{\text{rms}}^{\text{ABE}} = A_{\text{ABE}} \frac{e^2}{h} \sqrt{\frac{V_{\text{DC}}}{V_{c0}}} \left(\frac{L_\varphi(V_{\text{DC}})}{C_{\text{r}}} \right)^{0.65} e^{-\beta C_{\text{r}}/L_\varphi(V_{\text{DC}})}. \quad (2)$$

Good agreement of the ABE data with both eq. (2) and the UCF data is obtained for $\beta \approx 1.1$, as we show in fig. 2 by the solid curve and in fig. 3 by crosses.

Below we discuss how the observed power law dependences in $L_\varphi(V_{\text{DC}})$ can be linked to microscopic scattering processes. From figs. 2, 3 we can distinguish two regimes (denoted by I and II in fig. 3): (I) $\delta g_{\text{rms}}(V_{\text{DC}})$ increases with V_{DC} and $L_\varphi(V_{\text{DC}}) \propto V_{\text{DC}}^{-0.19}$, and (II), $\delta g_{\text{rms}}(V_{\text{DC}})$ decreases with V_{DC} while $L_\varphi(V_{\text{DC}}) \propto V_{\text{DC}}^{-0.57}$.

The phase-coherence length is determined by the dephasing rate τ_φ^{-1} via $L_\varphi = \sqrt{D\tau_\varphi}$. τ_φ^{-1} can be expressed as an average of the inelastic scattering rate $\tau_{\text{in}}^{-1}(\epsilon)$ (ϵ is the energy

exchanged in the interaction) of electrons over the accessible energy range given by the width of the electron distribution function f [15]. If the electron-electron scattering is dominant, the scattering rate is energy dependent as described by the kernel function $K(\epsilon) = \kappa_\eta \epsilon^{-\eta}$. The energy interval E accessible for the scattering is set by $k_B T$ in equilibrium and eV_{DC} in non-equilibrium. At high energies, where single scattering events determine τ_φ , a simple argument [15] leads to $\tau_{\varphi,ee}^{-1}(E) \propto E^{1/\eta}$ [20]. Two choices for η are currently under debate: $\eta = 3/2$ for the disorder-enhanced Coulomb interaction [15] and $\eta = 2$ for magnetic impurity-mediated interaction [16, 21]. This results in $L_{\varphi,ee}^{-1}(V_{\text{DC}}) \propto V_{\text{DC}}^{1/3}$ for $\eta = 3/2$, and $L_{\varphi,ee}^{-1}(V_{\text{DC}}) \propto V_{\text{DC}}^{1/4}$ for $\eta = 2$. Hence the kernel exponent $\eta = 2$ appears more consistent with our experimental observation of $L_\varphi^{-1} \propto V_{\text{DC}}^{0.19}$. This is supported by the observed saturation of the equilibrium τ_φ for temperatures below 1 K.

We emphasise that—independently of the precise interaction mechanism—the rather weak decrease of $L_{\varphi,ee}$ with increasing V is insufficient to suppress the LK enhancement of δg_{rms} . This is seen as follows. We recall that the condition for the appearance of the LK enhancement in a phase-coherent section of the wire is $V_{\text{DC}}^{\text{sec}} > V_c^{\text{sec}}$, which can be written as $V_{\text{DC}}/V_{c0} > (L_{c0}/L_\varphi)^3$. Since $L_{\varphi,ee} \propto V_{\text{DC}}^{-1/4}$, the right-hand side of the above inequality increases more slowly with V_{DC} than the left-hand side, and the condition for the enhancement is maintained over the whole voltage range of increasing $\delta g_{\text{rms}}(V_{\text{DC}})$.

While the regime I is determined by electron-electron scattering and a non-equilibrium electron distribution function f , the stronger decay of $L_\varphi(V)$ in the regime II is caused by electron-phonon scattering. It can be very well described by equilibrium properties if we assume local thermal equilibrium with an elevated electron temperature $T_{e1}(V_{\text{DC}})$. Electron-phonon scattering leads to $\tau_{\varphi,ep} \propto T_e^{-m}$, where m equals 3 for very clean samples, 4 for strongly disordered samples, and 2-3 for samples of intermediate degree of disorder [22]. Since T_e is related to the applied voltage by $T_e \propto V^{2/(2+m)}$ [23], one obtains $\tau_{\varphi,ep} \propto V^{-2m/(2+m)}$ and $L_{\varphi,ep} \propto V^{-m/(2+m)}$. Our result $L_\varphi(V_{\text{DC}}) \propto V_{\text{DC}}^{-0.57 \pm 0.03}$ gives $m = 2.6 \pm 0.3$, which agrees well with the value 2.5 obtained in a noise measurement on similar samples [24]. Moreover, our WL result ($L_{\varphi,ep}(T) \propto T^{-m/2}$) is $m = 2.4$, in excellent agreement with the above values. We also note that, even though T_e rises up to ~ 10 – 15 K at high V_{DC} , the energy averaging given by the LK approach remains essentially unaffected because T_e increases more slowly than the voltage itself. Although our phenomenological analysis leads to a consistent interpretation of our data, further theoretical work is needed to fully understand the dephasing in the case of strong non-equilibrium.

In conclusion, we have measured the universal conductance fluctuations and Aharonov-Bohm effect in mesoscopic gold samples under highly non-equilibrium conditions of large applied bias voltages. The rms fluctuation δg_{rms} of the differential conductance initially increases with voltage $\propto \sqrt{V/V_c}$, which demonstrates the validity of the theoretical prediction by Larkin and Khmel'nitskiĭ. This increase is followed by a decay of δg_{rms} at higher voltages, where inelastic scattering becomes substantial. The amplitude decays as a power law for the universal conductance fluctuations and exponentially in the case of the Aharonov-Bohm effect. The decrease of the phase-coherence length with increasing voltage is in good agreement with the inferred $L_\varphi(V)$ -dependences for electron-electron and electron-phonon scattering. In particular, the electron-electron collisions are not sufficient to suppress the enhancement mechanism.

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