Shot-noise: From Schottky's vacuum tube to present-day quantum devices

Christian Schönenberger and Stefan Oberholzer

Institute of Physics, University of Basel, Klingelbergstr. 82, CH-4056 Basel, Switzerland

ABSTRACT

Shot-noise in the electrical current through a 'device' is caused by random processes that determine the electron transport from source to drain. Two sources can be distinguished: on the hand, electrons may randomly emanate from the contacts (source and drain), because the relevant states in the reservoirs fluctuate. On the other hand, the transmission *through* the device is non-deterministic (non-classical). As we demonstrate in this article the former dominates noise in the vacuum tube, whereas the latter applies to coherent mesoscopic devices, which have been studied in great detail during the last decade.

Keywords: Shot-noise, charge transport, mesoscopic physics.

1. INTRODUCTION

Shot-noise in electrical devices is due to time-dependent fluctuations in the electrical current caused by the random transfer of discrete charge units. In 1918 Walter Schottky[1] analyzed these fluctuations in vacuum tubes for the first time arriving at his famous Schottky formula. It states that the spectral density S of the fluctuations at 'low' frequencies is proportional to the unit of charge e and to the mean electric current $|I|$, explicitly $S = 2e|I|$.

In recent years shot-noise of mesoscopic conductors has been investigated extensively.[2,3] In these systems shot-noise is a *quantum* phenomenon originating from diffraction of the wave function.[4] Responsible for shot-noise is the quantum mechanical uncertainty of not knowing with absolute certainty whether a particle incident on a scattering region will be transmitted from source to drain.

Schottky derived his formula before the existence of quantum mechanics, solely making use of classical statistical arguments. In retrospect, one may ask the question whether shot-noise in a vacuum tube is classical or not. The answer is not straightforward: Most engineers, for example, are convinced that shot-noise is a classical phenomenon altogether. The mesoscopic physics community, on the other hand, tend to believe that shot-noise in electrical conductors is quantum in general. This has motivated us to analyze the randomness contributing to shot-noise in vacuum tubes in detail. As we will show in this article, it turns out that quantum diffraction in the emission process can be neglected in vacuum tubes. The main source of noise stems from the *classical* occupation of electron states. (i.e. the Boltzmann tail) within the cathode. Hence, Schottky's vacuum tube is classical!

Before proceeding to an in-depth analysis of the source of noise of a vacuum tube, let us have a look at the prototype quantum system, which is a tunnel junction. In such a device, the stochastic nature of electron transfer originates from the quantum-mechanical uncertainty, which we describe by a transmission probability T. It appears like a coincidence that shot-noise in this device is given by Schottky's formula, too. Hence, although the nature of randomness is different, it is not apparent in the final formula for S, which remains to be $S = 2e|I|$.

In a macroscopic tunnel device, there may be several channels that contribute to transport and an appropriate average has to be considered. Shot-noise has, however, been measured in the smallest possible tunnel device, namely on a single atom contact realized by the tip of a scanning-tunneling microscope (STM).[5] This approach has two important advantages: First, the tininess ensures that only one channel contributes. Secondly, the electrical conductance of an STM junction is regulated at very low values as compared to the quantum conductance of an open channel, ensuring that the transmission probability T is very low. Typically, $T \sim 10^{-5}$. In fact, a small T is the requirement that 'full' shot-noise given by Schottky's formula is obtained. The measurement citeBirkPRL1995 indeed agrees nicely with Schottky's predication, see Fig. 1.

Send correspondence to: *christian.schoenenberger@unibas.ch*

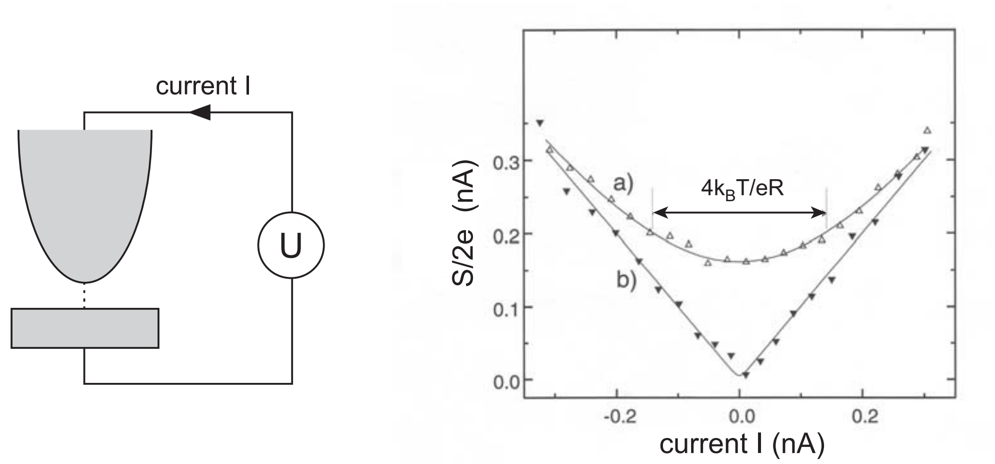

Figure 1. Left: schematics of an STM tip (Pt) held in close proximity to a clean metallic (Au) surface. Right: measured current noise S vs. current I, displaying the transition from thermal noise at equlibrium (i.e. for $I = 0$) to shot-noise a) at 300 K with $R \approx 0.32\,\text{G}\Omega$ (open triangles), and b) at 77 K with $R \approx 2.7\,\text{G}\Omega$ (solid triangles. The solid curves are theoretical predictions for 'full' shot-noise, corresponding to a slope of 1 in the non-equilibrium regime, i.e. $eV \gg k_BT$. $R = V/I$ is the tunneling resistance, which is stabilized by a feedback system during measurements. The feedback only responds to frequencies $\leq 10\,\text{kHz}$, whereas noise was measured at a much larger frequency of 200 kHz.

The present article is structured as follows. We briefly summarize the theoretical facts for shot-noise of mesoscopic devices in section 2. We introduce the parameters of a vacuum tube relevant for our analysis in section 3 and proceed to the analysis in section 4. Having studied the two most extreme cases for shot-noise in electrical devices, we briefly give an outlook into the power of shot-noise as a diagnostic tool in section 5 and close with an outlook in section 6.

2. SHOT-NOISE OF A TWO-TERMINAL CONDUCTOR

We start with a simple derivation of the expression for the power spectral density of the current noise of a two-terminal mesoscopic conductor along the lines of Martin and Landauer.[6] In their paper the fluctuating currents result from the random transmission of electrons from one terminal to the other. It is explicitly assumed that energy is conserved. Hence, there are no inelastic processes considered. Different processes contribute to the noise for each energy E and mode n:

1. A current pulse occurs whenever an electron wave packet incident from the left terminal is scattered into an empty state in the right terminal. The rate τ_{rl}^{-1} of these events is proportional to the probability $f_L(E)$ for an energy state E in the left reservoir to be occupied times the probability $1 - f_R(E)$ for the state in the right reservoir, in which the electron scatters into, to be unoccupied, times the transmission probability from left to right: $T_n^{rl}(E) \equiv T_n(E)$:

$$\tau_{rl}^{-1} \sim f_L(E)[1 - f_R(E)]\, T_n(E). \tag{1}$$

   The factor $1 - f_R(E)$ ensures that the Pauli principle is fulfilled.

2. Of course the reverse process, that electrons scatter from an occupied state in the right reservoir to an unoccupied state in the left reservoir, contributes to the noise too. The rate of these processes is given by:

$$\tau_{lr}^{-1} \sim f_R(E)[1 - f_L(E)]\, T_n(E), \tag{2}$$

   where $T_n^{lr}(E) = T_n^{rl}(E) \equiv T_n(E)$ has been taken into account.

Since we require an expression for the fluctuations, i.e. the deviations from the mean current, the mean current squared has to be subtracted. The mean current is proportional to $T_n(E)\,[f_L(E)-f_R(E)]$. Thus the contribution to noise from electrons at energy E in one specific mode n is proportional to:

$$f_L(E)[1-f_R(E)]\,T_n(E)+f_R(E)[1-f_L(E)]\,T_n(E) \\ -[f_L(E)-f_R(E)]^2\,T_n^2(E). \quad (3)$$

The yet undetermined prefactor follows from the fact that if no bias is applied ($V=0$) the expression for the thermal noise $4k_BTG$ must be recovered:

$$\begin{aligned} S &= 4k_BTG = 4k_BT\,G_0\int dE\left(-\frac{\partial f}{\partial E}\right)\sum_n T_n(E) \\ &= 2G_0\int dE\,2f(E)[1-f(E)]\sum_n T_n(E), \end{aligned} \quad (4)$$

with $G_0=\frac{2e^2}{h}$. In equilibrium ($V=0$) $f_L(E)=f_R(E)\equiv f(E)$. Thus the expression in (3) equals

$$2f(E)[1-f(E)]\,T_n(E). \quad (5)$$

Comparing Eqs. (4) and Eqs. (5) the prefactor to Eqs. (3) follows as $2G_0$. The general expression for shot-noise of a two-terminal conductor (in the zero frequency limit and without energy relaxation) follows as[7,8]:

$$\begin{aligned} S &= 2\,G_0\sum_n\int dE\,\{f_L(1-f_R)\,T_n+f_R(1-f_L)\,T_n \\ &\quad -[f_L-f_R]^2\,T_n^2\} \\ &= 2\,G_0\sum_n\int dE\,\{[f_L(1-f_R)+f_R(1-f_L)]\,T_n(1-T_n) \\ &\quad +[f_L(1-f_L)+f_R(1-f_R)]^2\,T_n^2\} \end{aligned} \quad (6)$$

This final equation has two terms, one containing the transmission probabilities in the combination $T_n(1-T_n)$ and the other squared, i.e. T_n^2. In general both contribute, but there are limiting cases. For example, at zero temperature, the prefactor of the second term is zero and S is proportional to the applied bias and to the sum of $T_n(1-T_n)$. Another limit is given by a deterministic system for which transmission is either zero or one. In this case, the first term is zero and S is solely given by the fluctuation of the electrons within the two reservoirs, which is expressed by the products $f_{L,R}(1-f_{L,R})$. It is clear from these two limits that the stochastic nature of electron transmission, which is caused by quantum mechanical diffraction, is contained in the first term.

3. VACUUM TUBES

Figure 2(a) shows a schematics of a vacuum tube (triode): The heated cathode (K) made of a wounded tungsten wire boils off electrons into vacuum. These are attracted by the positively charged anode (A) (Edison effect). A grid (or many grids) between cathode and anode, which is negatively charged, controls the electron current. By designing the cathode, grid(s) and plate properly, the tube will convert a small AC signal voltage into a larger AC voltage, thus amplifying it.[9]

In case that the anode is floating no net current will flow from the cathode to the anode [Fig. 3(a)]. Instead a negative space-charge is formed in front of the cathode, originating from evaporated electrons which are hold back by the ionized atoms. The size χ of the space-charge region can be calculated solving the Poisson-equation $\Delta\varphi(x)=-en(x)/\epsilon_0$ for the electrical potential $\varphi(x)$ with the electron density $n(x)=n_0\exp(-e\varphi(x)/k_B\theta)\simeq n_0\,[1-e\varphi(x)/k_B\theta]$, where n_0 is the electron densitiy within the cathode, θ the temperature, and ϵ_0 the dielectric constant of the vacuum:

$$\chi=\sqrt{\frac{\epsilon_0k_B\theta}{n_0e^2}}. \quad (7)$$

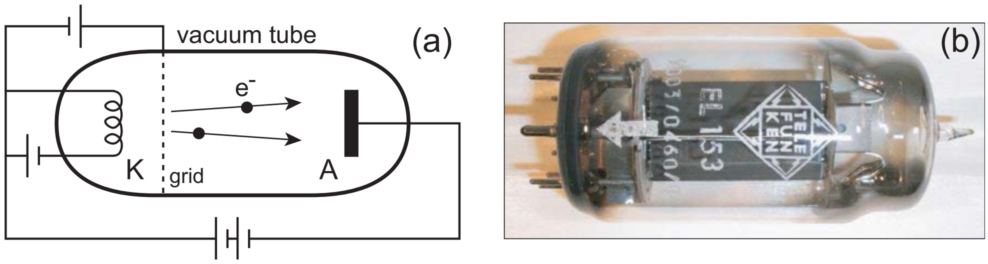

Figure 2. Vacuum tubes: (a) Schematics of a triode. Electrons having energies larger than the work function W of the tungsten filament are emitted from the heated cathode (K), travel through the vacuum and are attracted by the positive anode (A). (b) Photograph of a historical tetrode (triode with additional grid) containing 4 electrodes (Telefunken EL 153).

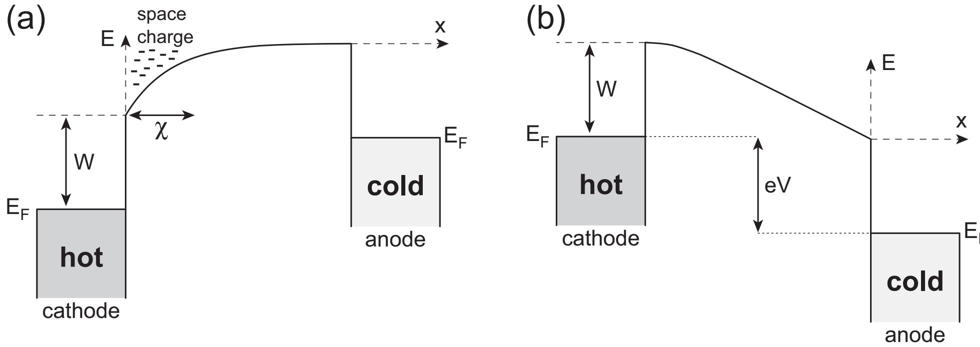

Figure 3. (a) Space-charge region formed in front of the cathode in a open-circuited tube. (b) For sufficiently high bias voltages V the space-charge is removed (saturation regime) and the potential drops linearly. W denotes the work function.

The higher the temperature the larger the space-charge region.

When the circuit is closed and the cathode is kept at a higher temperature than the anode, a thermionic current will flow from the cathode to the anode. The magnitude of this current is limited by the negative space-charge region in front of the cathode. This is also true when the anode is kept at a (moderate) positive potential with respect to the cathode, because the large number of electrons near the cathode can effectively screen the field due to the anode. In this space-charge limited regime, the current is given by

$$I = \frac{\sqrt{2}}{9\pi}\sqrt{\frac{e}{m}}\frac{V^{3/2}}{L^2} \tag{8}$$

with L the distance between cathode and anode.[9] Only when the bias voltage V is sufficiently large all electrons are attracted by the anode and the space-charge region is removed [Fig. 3(b)]. In this case, the current saturates (does no longer depend on the anode voltage) and is determined by the temperature of the cathode [Fig. 4].

In the space-charge limited regime, where the possibility of escape of an electron is strongly hindered by the Coulomb repulsion, shot-noise is suppressed. Full shot-noise $S = 2e|I|$ is only present in the saturation regime.[10] The question whether the shot-noise in the saturation regime is classical or quantum in nature is discussed in Sec. 4. Before, the electrical field and current in the saturation regime will be determined.

3.1. Electrical field and current in the saturation regime

At the edge of the vacuum barrier the electron density is approximatively given by $n_0 = a^{-3}\exp(-W/k_B\theta) \simeq 5\cdot 10^{16}\,\mathrm{m}^3$ with $a \simeq 1.2\,\mathring{A}$ the typical interatomic distance, $W = 4.5\,\mathrm{eV}$ the work function of tungsten and the cathode temperature $\theta = 2000\,\mathrm{K}$. The size χ of the space-charge region follows from (7) and is of the order

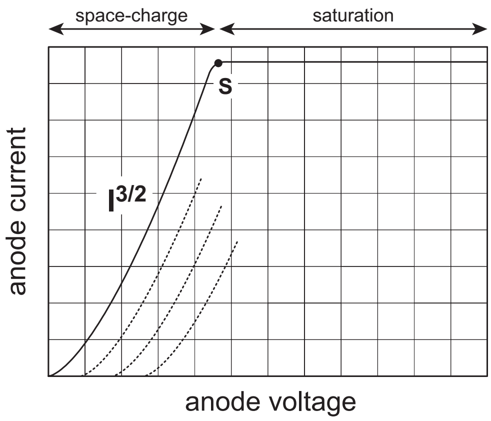

Figure 4. Current-voltage characteristics of a vacuum tube illustrating the 3/2-power law in the space-charge regime [Eqs. (8)] and the saturation point (S). The dashed curves are IV-curves for different grid voltages. Within the saturation regime the current does not depend on the anode voltage because all electrons emitted by the cathode are collected at the anode.

$10\,\mu$m. The charge build up at the cathode corresponds to an electrostatic surface potential of $k_B\theta/e$ so that the surface electric field can be estimated as $\mathcal{E} \simeq k_B\theta/e\chi$. Inserting numbers the *saturation field* is of the order 10^4 V/m.

The electrical current density due to thermionic emission from a heated conductor is given by the Richardson-Dushman equation[11]:

$$j = \mathcal{L}\,\theta^2 \exp(-W/k_B\theta) \tag{9}$$

with $\mathcal{L} = emk_B^2/2\pi^2\hbar^3 120\,\mathrm{AK^{-2}cm^{-2}}$. This expression is only correct if the electrical field $\mathcal{E}$ is high enough so that the space-charge is removed (saturation regime). The saturation field $\mathcal{E}$ combined with the potential formed by the image charges in the (planar) cathode leads to an electrical potential $\phi(x)$ given by

$$\phi(x) = -\mathcal{E}x - \frac{e}{4\pi\epsilon_0}\frac{1}{x}, \tag{10}$$

which is illustrated in Fig. 5. The maximum of $\phi(x)$ lies at $x_0 = \sqrt{e/4\pi\epsilon_0\mathcal{E}}$, where the barrier is lowered by $e\phi(x_0) = -2e\sqrt{e\mathcal{E}/4\pi\epsilon_0} \simeq 8\,\mathrm{meV}$. This is negligible in comparison with the work function $W = 4.5\,\mathrm{eV}$, so that the saturation current can be estimated disregarding the barrier lowering. For a cathode area of $10^{-2}\,\mathrm{cm}^{-2}$ and $\theta = 2000\,\mathrm{K}$ the *emission current* is of the order $10\,\mu$A.

4. THE 'SCHROTEFFEKT' IN VACUUM TUBES

In the saturation regime, where no space-charge region exists at the cathode and $f_R = f_{anode} = 0$, the shot-noise power due to the emission of electrons from the cathode is according to Eqs. (6) given by

$$S = 2G_0 \sum_n \int dE\,\{\underbrace{f_{cathode}T_n(1-T_n)}_{quantum} + \underbrace{f_{cathode}\,T_n^2}_{classical}\}. \tag{11}$$

Here we have made use of the fact that the occupation of the hot cathode is small (classical): $f_L = f_{cathode} = \exp(-E/k_B\theta) \ll 1$. Therefore $f_L(1-f_L) \simeq f_L$. The current I due to emission at the cathode equals

$$I = \frac{2e}{h}\sum_n \int dE\, f_{cathode}\,T_n \tag{12}$$

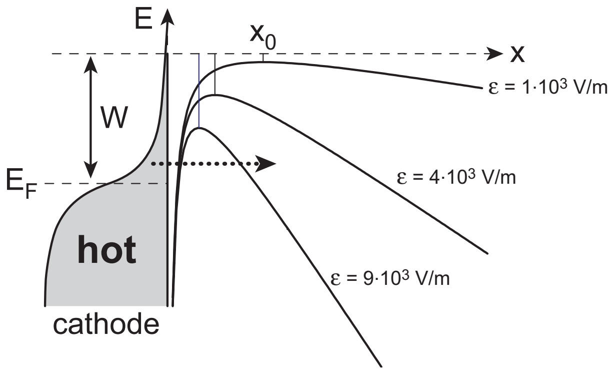

Figure 5. Image potential determining the barrier shape in emission of electrons from the hot cathode for different electrical fields $\mathcal{E}$. For very high fields the barrier becomes very thin so that electrons can tunnel through and the emission noise will be of quantum nature.

There are two terms in Eqs. (11) contributing to the noise: the first term is the quantum mechanical part since it only contributes for transmission probabilities $T \neq \{0, 1\}$. The second term is classically because it dominates when the transmission process is deterministic, i.e. $T_n = 0$ or 1. Let us evaluate S in the two limiting cases:

Classical: Because all T_n's are either 0 or 1, $T_n^2 = T_n$ and shot-noise is given by

$$S = 2G_0 \sum_n \int dE\, f_{cathode}\, T_n. \tag{13}$$

The current is

$$I = \frac{2e}{h} \sum_n \int dE\, f_{cathode}\, T_n \tag{14}$$

so that the Fano factor $F \equiv S/2e|I|$ follows as

$$F = \frac{1}{2e} \frac{4e^2}{h} \frac{h}{2e} = 1 \tag{15}$$

which is Schottky's formula, i.e. $S = 2e|I|$.

Quantum: In this limit there exists T_n's that are neither zero nor one. We consider here only the case where all T_n's are small ($T_n \ll 1$), which corresponds to the tunneling limit. In this case the quantum term in Eqs. (11), which is $\propto T_n$, dominates, whereas terms proportional to T_n^2 are negligibly small. S is then given by

$$S = 2G_0 \sum_n \int dE\, f_{cathode}\, T_n. \tag{16}$$

For the current the same expression (14) as in the classical case holds, resulting in a Fano factor of 1 as before. We again obtain Schottky's formula $S = 2e|I|$, this time however, originating from quantum diffraction.

Thus, in order to decide wether shot-noise in a vacuum tube is classical or quantum, one has to evaluate the transmission probabilities T_n through the 'device'. If the current is carried exclusively by channels with $T_n = 1$, quantum diffraction does not matter and shot-noise is classical.

4.1. Transmission probability at the cathode

The quantum-mechanical transmission probability of electrons with energy E above the barrier [Fig. 6] can be determined from[12]

$$T \simeq \left[1 + e^{-2\pi E/\hbar\omega_0}\right]^{-1} \tag{17}$$

[h]

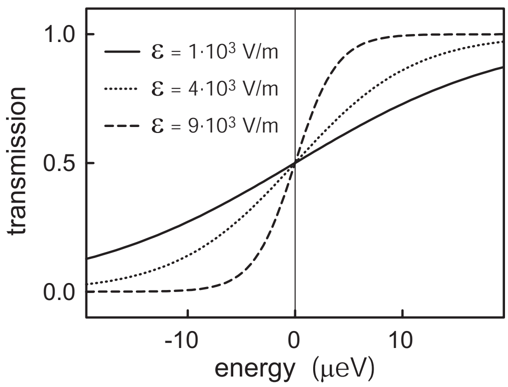

Figure 6. Transmission probability vs. energy according Eqs. (17) for different saturation fields $\mathcal{E}$.

The energy E is given by $k_B\theta$ with θ the cathode temperature. ω_0 denotes the negative curvature at the barrier top and determines whether the barrier is sharp or smooth. It can be obtained from the 'force-constant'

$$f = e\left|\frac{\partial^2\varphi}{\partial x^2}\right|_{x=x_0} = 2\sqrt{4\pi\epsilon_0 e}\cdot\mathcal{E}^{3/2} \tag{18}$$

with $\omega_0 = \sqrt{f/m}$[13]:

$$\omega_0 = \left(\frac{16\pi\epsilon_0 e}{m^2}\right)^{1/4}\cdot\mathcal{E}^{3/4}. \tag{19}$$

If $\hbar\omega_0 \ll k_B\theta$, $T = 1$ and the classical part of the shot-noise in Eqs. (11) dominates. In the other limit $\hbar\omega_0 \gg k_B\theta$ the transmission T is very small and shot-noise is due to tunneling (quantum diffraction).

First rough shot-noise measurements in vacuum tubes were carried out by Hartmann in 1921.[14] A very careful study of the 'Schroteffekt' was performed by Hull and Williams in 1925.[10]

In the first part of the latter experiment shot-noise was measured in the saturation regime, where the thermionic current is limited by temperature.[15] The corresponding parameters are given in the first two lines of Tab. 1. In this regime full Schottky-noise $2e|I|$ ($F = 1$) has been measured in excellent agreement with predictions. The ratio $\hbar\omega_0/k_B\theta \ll 1$ so that the transmission is 1. Therefore, shot-noise in this experiment is *classical*.

In the second part of the experiment the effect of the space-charge on the shot-noise was investigated at lower electric fields $\mathcal{E}$. The corresponding parameters are given in the last three lines of Tab. 1. At lower cathode temperatures full Schottky-noise is still observed. At higher temperatures, however, the space-charge region builds up and disturbs the free transmission of electrons from cathode to anode. Shot-noise is gradually suppressed due to Coulomb interaction. The space-charge region can be seen as a large capacitor which effectively shunts the fluctuations. A similar effect occurs in a classical ohmic wire, in which inelastic scattering effectively 'damps' the fluctuations. This is why a classical resistor displays only equilibrium noise, but no shot-noise.

In summary, shot-noise in a vacuum tube is classical. It is solely determined by the Boltzmann tail in the occupation of states of the cathode that are allowed to emit. Diffraction from the cathode to the anode is negligibly. To our knowledge, this analysis is new. It may be of academic interest only, but since today Schottky's formula goes much beyond the historical derivation it has been a worth exercise. There are much more devices which display conventional shot-noise, but look less classical than a 2000 K hot vacuum tube. One may ask the same question for the reverse saturation current of a diode. Is S classical or rather quantum. W e are not aware of any analysis for this device and leave this open for the future.

$\mathcal{E}$ [V/m]	V_G [V]	V_P [V]	i_0 [mA]	θ [K]	$\hbar\omega_0/k_B\theta$	T	F
$3 \cdot 10^6$	120	120	1	1675	$3.2\,10^{-2}$	1	1.00
$3 \cdot 10^6$	120	120	5	1940	$2.7\,10^{-2}$	1	1.00
$1 \cdot 10^4$	-6	130	1	1675	$4.4\,10^{-4}$	1	0.93
$1 \cdot 10^4$	-6	130	3	1805	$4.1\,10^{-4}$	1	0.49
$1 \cdot 10^4$	-6	130	5	1940	$3.8\,10^{-4}$	1	0.20

Table 1. Experimental parameters from shot-noise measurements of Hull and Williams in 1925. V_G is the voltage at the grid and V_P at the anode plate. i_0 is the thermionic current. $F = S/2e|I|$ denotes the Fano factor. The second last column shows that the shot-noise observed in this experiment is a classical phenomenon.

5. SHOT-NOISE AS A DIAGNOSTIC TOOL IN MESOSCOPIC DEVICES

Mesoscopic circuits are usually operated at low temperatures and shot-noise is measure under strongly non-equilibrium conditions, meaning $eV >> k_B\theta$, where V is the applied voltage and θ the temperature. If this holds one can set the temperature to zero in calculations to first order.[16] The low temperatures should further ensure that the transport is elastic and coherent.[3] Following our discussion, the noise of the connecting leads can now be disregarded and shot-noise is determined by the quantum-mechanical diffraction through the device alone. S is proportional to $\sum T_n(1 - T_n)$ and henceforth, the Fano factor is given by $F = \langle T_n(1 - T_n)\rangle/\langle T_n\rangle$, because the conductance $G \propto \sum T_n$. Here $\langle\cdots\rangle$ denotes the average over channels. In a single-channel quantum wire, the Fano factor is simplified and given by $F = (1 - T)$. Consequently, shot-noise is suppressed below the Schottky value $S = 2e|I|$. This suppression is *generic* for electron systems and originates from the Pauli exclusion principles. In fact, a perfect quantum wire, for which $T = 1$, is 'silent'.[17,18] Noise is absent because the transmission process is deterministic in this limit. An electron that enters the wire from the source contact will be transmitted to the drain with certainty. It is quite amusing that a perfect quantum wire can be considered classical in this respect!

The situation is more exciting for systems that have many channels, for example a chaotic cavity or a diffusive (but coherent) wire. Assume we know the number of channels that contribute to electrical transport, we can determine the mean transmission probability $\langle T_n\rangle$ from G and the variance from S. Taking a diffusive wire as an example, one may naively expect that all 'eigenchannels' have a small T_n, because scattering is homogenously distributed in the wire. In fact, the mean transmission probability $\langle T_n\rangle$ is in general small. If all T_n's were small too, shot-noise would again be full and given by Schottky's formula. However, this is not the case. Shot-noise is suppressed by a universal factor which only depends on the system but not on the specific realization. The Fano factor is 1/3[19,20] for a diffusive wire and it is 1/4[3,21,22] for a chaotic cavity. These results show that the transmission eigenvalues are not uniformly distributed. Instead, the distribution is bimodal and strongly peaks at $T = 0$ and $T = 1$.[23] Although there is diffusion, there are always channels that are almost open. These are the ones that carry most of the current, but do not contribute to shot-noise. There are also almost closed channels that do not contribute to current. Finally, there are a fraction of channels with intermediate transmission probabilities that determine shot-noise. The surprising fact of bimodal distributions can be seen as one hallmark of theory in mesoscopic physics during the last decade.[23] In studies of charge transport through mesoscopic multichannel systems, shot-noise has been used as a diagnostic tool. The predicted suppression factors have been confirmed[22,24,25]

To summarize, shot-noise in an electronic system, in which inelastic processes are absent, is in general suppressed due to the existence of open channels with $T_n \sim 1$. However, this is not the full story. Shot-noise may show an additional suppression, if the transmission process becomes classically deterministic. Consider a chaotic cavity. If the diffraction of the bouncing electron paths are large, a universal Fano factor of $F = 1/4$ is obtained.[3,21,22] However, diffraction may be week so that classical trajectories are a good approximation. Then, shot-noise should disappear because a classical trajectory, no matter how complicated it is, either ends at the drain or source contact.[4,26] Hence, there is no uncertainty. We have performed such an experiment in a cavity, which could be tuned from quantum to classical and demonstrated that F is indeed suppressed below

$F = 1/4$.[27] This experiment proves that shot-noise in mesoscopic devices is a quantum phenomenon and clearly distinct to noise in a vacuum tube.

6. OUTLOOK

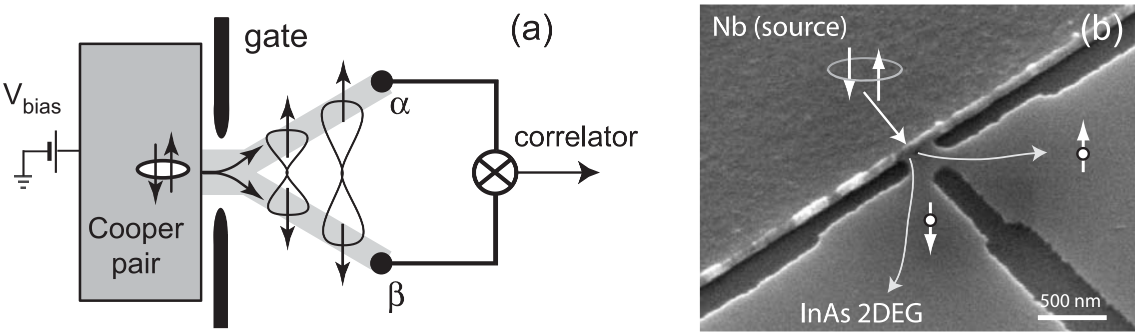

Figure 7. (a) Possible scheme of an entangler based on a superconductor as an injector and a high-mobility 2-dimensional electron gas (2DEG) beam-splitter (Y-junction). (b) An actual realization.[55]

In recent years, different kind of sources of entangled electrons in solid-state devices have been considered theoretically.[28–36] Such sources would allow to study '2nd order' correlations in nanostructure. Also here, shot-noise is of great value as a diagnostic tool.[37–42] Proposals range from quantum dots, carbon nanotubes, ring structures and edge states. The majority of proposals rely on 'natural' entanglement provided by Cooper pairs that form the ground state of a superconductors. At bias voltages smaller than the gap energy (subgap regime), only Cooper pairs can be transported from a superconductor, acting as source, trough a tunneling barrier into a normal region. One now has to invent means to split the spin-entangled pairs into two different orbital wave functions. In practice these wave functions correspond to two different exit channels through which the two electrons will escape [Fig. 7]. Using shot-noise, one can first prove that pairs are injected because this should enhance (ideally double) the overall noise.[43–47]

Secondly, one can then perform a Hanbury-Brown Twiss like correlation measurement[48,49] on the two exit leads to test the degree of charge separation. Whereas in a single-electron picture, correlations are always negative (antibunching) in an electronic device, the correlation signal is expected to become positive if two electrons are injected simultaneously and leave the device through different exit leads. Positive correlations are a signature of bosonic behaviour. As has been shown by Hanbury-Brown and Twiss (HBT), the correlations are positive for a stream of particles obeying Bose-Einstein statistics, e.g. for photons that are emitted from a thermal light source.[50] Also, it is obvious, that if the spin-singlet Cooper pair splits up in a Y-junction the way indicated in Fig. 7, the measured correlations ought to be positive. The opposite is however not true. One cannot directly conclude from measured positive correlations that one has realized a solid state entangler. It is possible, for example, that the spin-correlation is lost, due to spin-orbit interaction. There are more subtle scenario that support this statement.[51,52] In a well defined single-channel collision experiment on an electron beam splitter, it has theoretically been shown that the measured correlations are sensitive to the spin entanglement.[39]

On the experimental side, we are well behind the optical community, where single-photon sources exits in which photons behave as electrons, where entangled pairs of photons can be routinely produced and are already exploited for secure data communication. First steps in the realization of sources of spin-entangled electrons in the solid state have however been done. A collision experiment on a beam splitter was performed,[53,54] HBT-like correlation experiments have been demonstrated,[48,49] and enhanced shot-noise in superconducting-normal metal systems has been found.[43–47] What would be required in the future are tunable semiconducting heterostructures, in which quantum dots, spin filters and the like can be integrated together with highly transparent superconducting contacts. Altogether, this is a formidable task.

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REFERENCES

1. W. Schottky *Ann. Phys. (Leipzig)* **57**, p. 541, 1918.
2. C. W. J. Beenakker and C. Schönenberger *Physics Today* **56**, p. 37, 2003.
3. Ya. M. Blanter and M. Büttiker *Phys. Rep.* **336**, p. 1 2000.
4. C. W. J. Beenakker and H. van Houten *Phys. Rev. B* **43**, p. 12066, 1991.
5. H. Birk, M. J. M. de Jong, and C. Schönenberger *Phys. Rev. Lett.* **75**, p. 1610, 1995.
6. T. Martin and R. Landauer *Phys. Rev. B* **45**, p. 1742, 1992.
7. G. B. Lesovik *JETP Lett.* **49**, p. 592, 1989.
8. M. Büttiker *Phys. Rev. Lett.* **65**, p. 2901, 1990.
9. L. R. Koller, *The Physics of Electron Tubes*, McGraw-Hill, New York, 1937.
10. A. W. Hull and N. H. HWilliams, *Phys. Rev. B* **25**, p. 147, 1925.
11. A. Sommerfeld and H. Bethe, *Elektronentheorie der Metalle*, Springer Verlag, Heidelberg, 1967.
12. L. D. Landau and E. M. Lifschitz, *Quantum Mechanics, Nonrelativistic Theory*, Pergamon Press, Oxford, 1965.
13. This is analogous to the mechanical oscillator $\ddot{x} + \omega_0^2 x = 0$ with $F = -fx$ and $E_{pot} = \frac{1}{2} f x^2$.
14. C. A. Hartmann *Ann. Phys.* **65**, p. 51, 1921.
15. In order to avoid space-charge a very strong electric field at the cathode was achieved by connecting the anode and grid together.
16. In an actual experiment at low temperatures it may be a quite difficult task to ensure that the reservoirs to not heat up.
17. M. Reznikov, M. Heiblum, H. Shtrikman and D. Mahalu *Phys. Rev. Lett.* **75**, p. 3340, 1995.
18. A. Kumar, L. Saminadayar and D. C. Glattli *Phys. Rev. Lett.* **76**, p. 2778, 1996.
19. C. W. J. Beenakker and M. Büttiker *Phys. Rev. B* **46**, p. 1889, 1992.
20. K. E. Nagaev, *Phys. Lett. A* **169**, p. 103, 1992.
21. R. A. Jalabert, J.-L. Pichard and C. W. J. Beenakker *Europhys. Lett.* **27**, p. 255, 1994.
22. S. Oberholzer, E. V. Sukhorukov, C. Strunk, C. Schönenberger and T. Heinzel *Phys. Rev. Lett.* **86**, p. 2114, 2001.
23. C. W. J. Beenakker *Rev. Mod. Phys.* **69**, p. 731, 1997.
24. A. Steinbach, J. M. Martinis and M. H. Devoret *Phys. Rev. Lett.* **76**, p. 2778, 1996.
25. M. Henny, S. Oberholzer, C. Strunk and C. Schönenberger *Phys. Rev. B* **59**, p. 2871, 1999.
26. O. Agam, I. Aleiner and A. Larkin *Phys. Rev. Lett.* **85**, p. 3453, 2000.
27. S. Oberholzer, E. V. Sukhorukov and C. Schönenberger *Nature* **415**, p. 765, 2002.
28. P. Recher, E. V. Sukhorukov and D. Loss *Phys. Rev. B* **63**, p. 165314, 2001.
29. G. B. Lesovik, T. Martin and G. Blatter *Eur. Phys. J. B* **24**, p. 287, 2001.
30. P. Recher and D. Loss *Phys. Rev. B* **65**, p. 165327, 2002.
31. C. Bena, S. Vishveshwara, L. Balents and M. P. A. Fisher *Phys. Rev. Lett.* **89**, p. 037901, 2002.
32. N. M. Chtchelkatchev, G. Blatter, G. B. Lesovik and T. Martin *Phys. Rev. B* **66**, p. 161320, 2002.
33. V. Bouchiat, N. Chtchelkatechev, D. Feinbarg, G. B. Lesovik, T. Martin and J. Torrès *Nanotechnology* **14**, p. 77, 2003.
34. A. Crépieux, R. Guyon, P. Devillard and T. Martin *Phys. Rev. B* **67**, p. 205408 , 2003.
35. P. Samuelsson, E. V. Sukhorukov and M. Büttiker *Phys. Rev. Lett.* **91**, p. 157002, 2003.
36. C. W. J. Beenakker, C. Emary, M. Kindermann and J. L. van Velsen *Phys. Rev. Lett.* **91**, p. 147901, 2003.
37. J. Torrès and T. Martin *Eur. Phys. J. B* **12**, p. 319, 1999.

38. D. Loss and E. V. Sukhorukov *Phys. Rev. Lett.* **84**, p. 1035, 2000.
39. G. Burkard, D. Loss and E.V. Sukhorukov, *Phys. Rev. B* **61**, p. R16303, 2000.
40. J. Börlin, W. Belzig and C. Bruder *Phys. Rev. Lett.* **88**, p. 197001, 2002.
41. P. Samuelsson and M. Büttiker *Phys. Rev. Lett.* **89**, p. 046601, 2002.
42. M. Büttiker and P. Samuelsson *Physica E* **18**, p. 60, 2003.
43. P. Dieleman, H. G. Bukkems, T. M. Klapwijk, M. Schicke and K. H. Gundlach *Phys. Rev. Lett.* **79**, p. 3486, 1997.
44. X. Jehl, P. Payet-Burin, C. Baraduc, R. Calemczuk and M. Sanquer *Phys. Rev. Lett.* **83**, p. 1660, 1999.
45. X. Jehl, M. Sanquer, R. Calemczuk, and D. Mailly *Nature* **405**, p. 50, 2000.
46. A. A. Kozhevnikov, R. J. Schoelkopf and D. E. Prober *Phys. Rev. Lett.* **84**, p. 3398, 2000.
47. F. Lefloch, C. Hoffmann, M. Sanquer and D. Quirion *Phys. Rev. Lett.* **90**, p. 067002, 2003.
48. M. Henny, S. Oberholzer, C. Strunk, T. Heinzel, K. Ensslin, M. Holland and C. Schönenberger *Science* **284**, p. 296 (1999).
49. W. D. Oliver, J. Kim, R. C. Liu and Y. Yamamoto *Science* **284**, p. 299 (1999).
50. R. Hanbury Brown and R. Q. Twiss *Nature* **177**, p. 27, 1956 and **178**, p. 1046, 1956.
51. C. Texier and M. Büttiker *Phys. Rev. B* **62**, p. 7454, 2000.
52. A. Martin and M. Büttiker *Phys. Rev. Lett.* **84**, p. 3386, 2000.
53. C. Liu, B. Odom, Y. Yamamoto and S. Tarucha *Nature* **391**, p. 263, 1998.
54. R. C. Liu, Y. Yamamoto and S. Tarucha *Physica B* **249**, p. 152, 1998.
55. B. R. Choi and A. E. Hansen *et al.*, to be published.